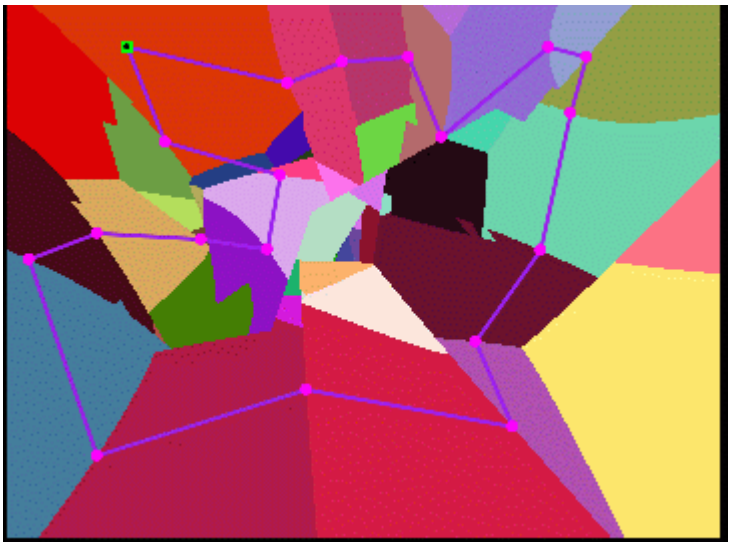
Traveling Salesman's SketchPad 1.0
TSSP

Tutorial Addendum



Understanding Isomorphic Regions

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What are Equal Permutation Regions?

1) Imagine that there are 8 cities joined by a Tour. (In the four examples below, the Tour is always optimal, i.e. the algorithm exhaustively searches and finds the shortest possible such Tour through all the cities. However the idea can be applied to any other algorithm generated Tour)

2) Imagine that 7 of the cities (colored magenta) are fixed in location (as any good old city should be) but the 8th city (colored green) is "movable".

3) For any fixed location of the "green city" we will have an optimal Tour via the algorithm.

4) Imagine that now you move the "green city", slowly at first.

5) The optimal Tour stays initially the same, however at one point the Tour will change. That is, if, initially the Tour went through the cities (say named A,B,C,D,E, F,G) in the permutation order: ABCDEFGH, now it may go in the permutation order: ACBDEFHG

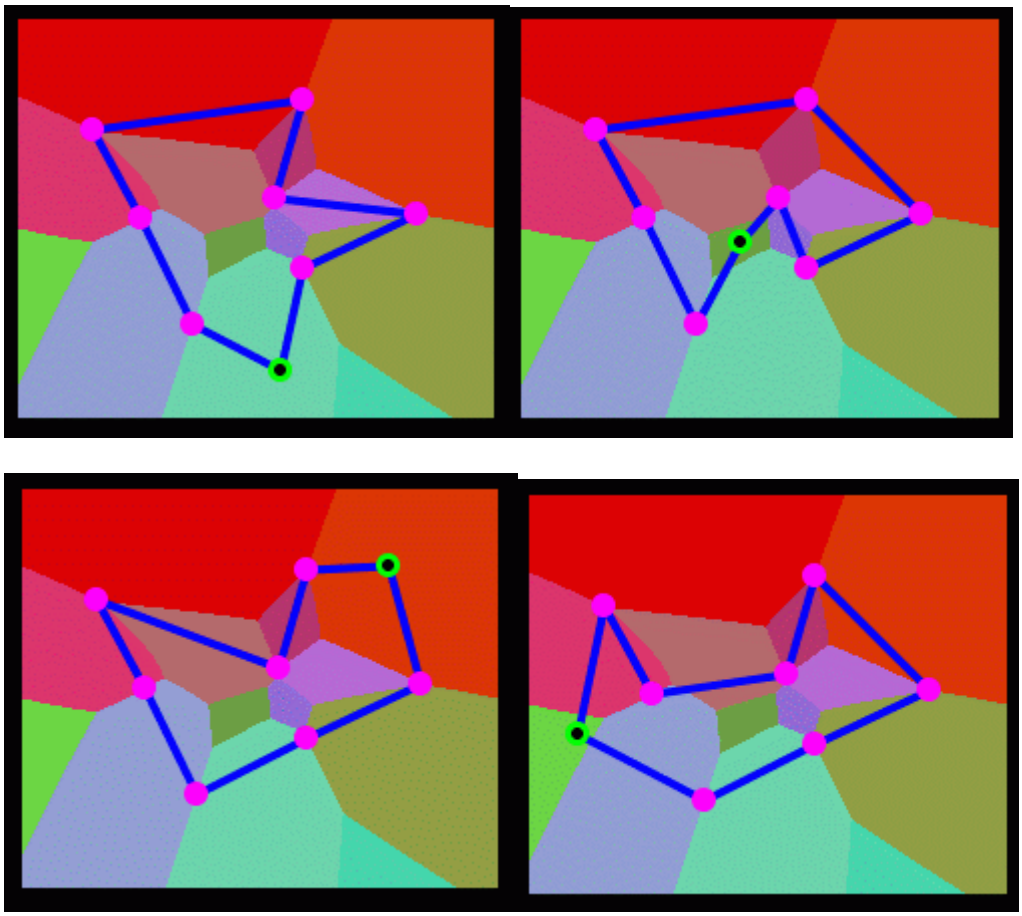
6) Now imagine placing the "green city" at every location of a rectangular region, finding the optimal Tour permutation and then coloring that location with a unique color for the permutation found.

7) Equal permutations get the same color and different ones get different colors

8) In TSSP the movable "green city" is called the "probe Node".

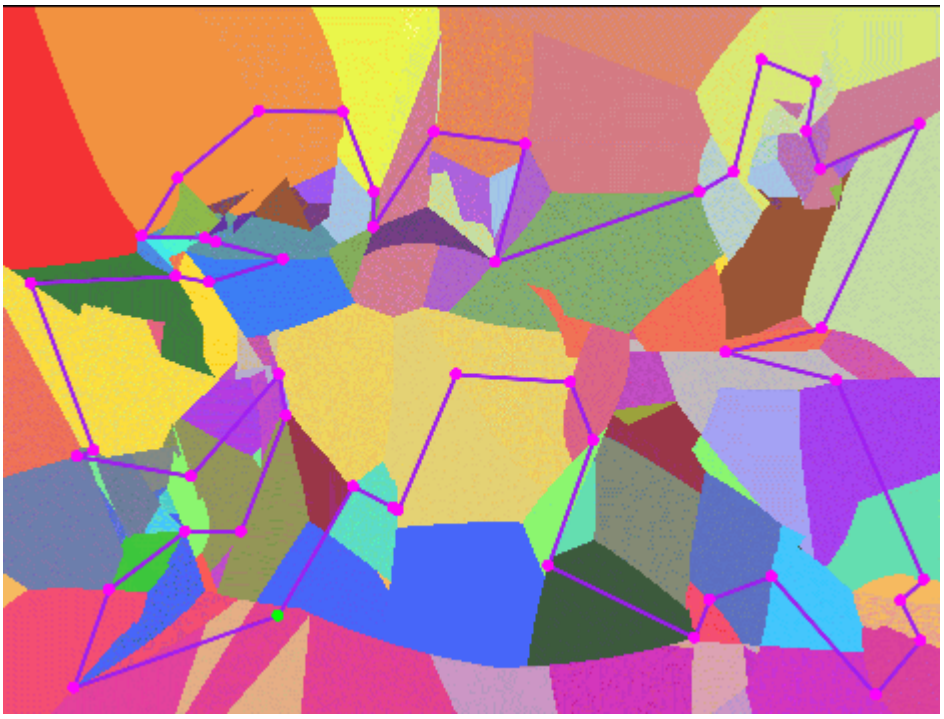
9) When you play with TSSP you will notice that when moving the "probe Node" within a given colored region the Tour maintains a particular permutation order of visiting the Nodes, where as when moving the "probe Node" across to another colored region yields another (sometimes a drastically different) permutation.

10) Studying the images below and then playing with TSSP should give you a feel for the concept discussed here.



You may wonder, "so what?" What is the use of these regions? In real world there are no "movable cities" (except may be during a quake and unfortunately fewer and fewer of them are "green " nowadays), so what is the point?

Honestly, I am not sure of the answer. I pursued this idea initially because it yielded interesting images, especially for non-optimal algorithms such as the MST-Tour algorithm. But later I came realize that these colored regions are in a sense, regions of "permutational stability" of a Tour (dependent of course on a given algorithm and the other initial conditions). So, if an image has a lot of colors



(see the TSSP Image Gallery Document for many examples) that means that the Tour is not very stable under its algorithm and altering the initial conditions slightly (via the movement of the "green city") can yield drastically different resulting Tours. Such a concept reminds me of the "butterfly effect" and wonder about its possible place in the Traveling Salesman Problem (TSP).

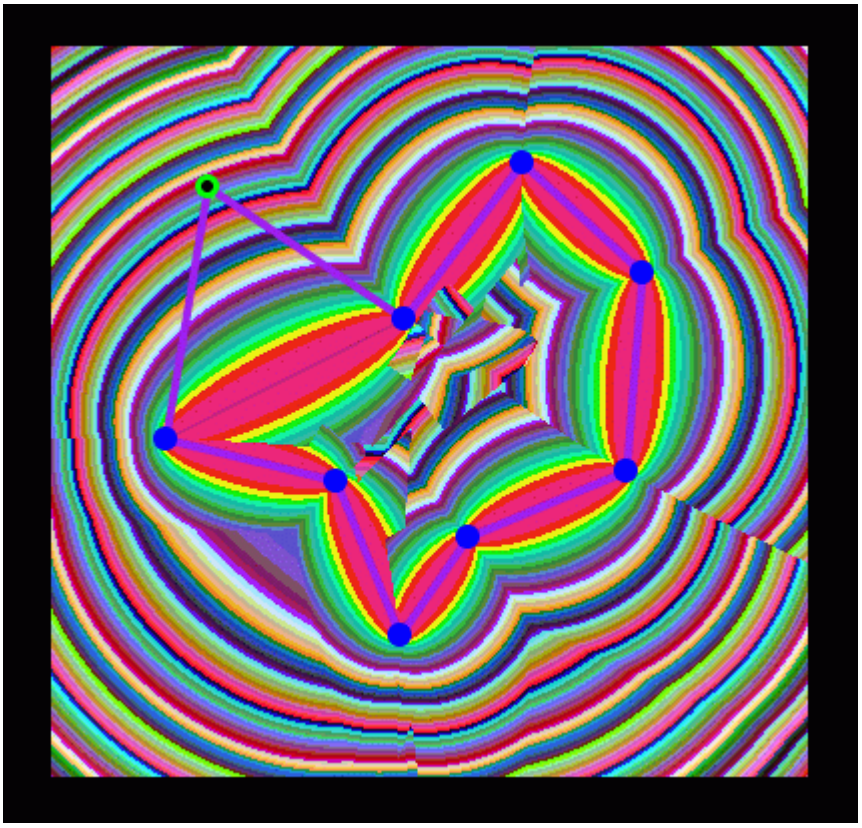
Below are three instances of the same data, with only minor change in the location of the "green probe Node". Notice how the resulting Tours are quite different.


cheapest insertion tour length = 1835.442


cheapest insertion tour length = 1881.462


cheapest insertion tour length = 1914.678

What are Equal Perimeter Regions?

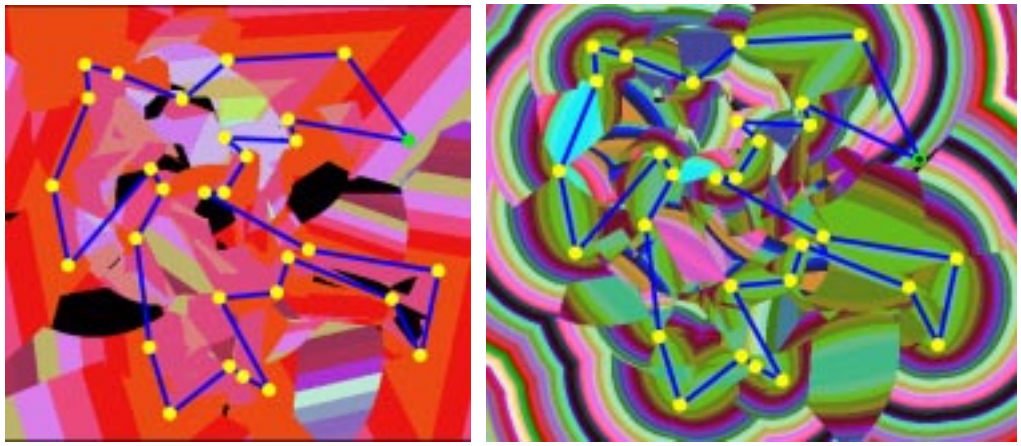


These are regions within which, moving the "probe Node" maintains a particular perimeter range for the Tour. This perimeter range is denoted as "bandwidth" in TSSP. For example if the "bandwidth" was 10 for the following image, then if we moved the "green probe Node" say within the red region, the perimeter of the Tour will stay within, say, the range 250-260 units.

What are Equal Area Regions?

The idea behind Equal Area Regions is very similar to that of Equal Perimeter Regions except the measurement involves Tour area rather than Tour perimeter. Note that the resulting images are very different. Compare these images for the same data. (In the first pair we have a "red probe Node" for a change.) The Tour algorithm used here for both examples is "Cheapest Insertion". The left images are Equal Area Regions and the right ones are Equal Perimeter Regions.

Another example:



Note: For Equal Area Regions black areas occur when Tour is self-crossing and therefore has no meaningful area defined.