An Analysis of the Trio Duel Puzzle

by Selim Tezel in collaboration with Sarah Thornton

Our thanks go to Mr. Gary Hawley for informing us about this lovely puzzle that appeared on the NPR Car Talk show

Who's Shootin' Who?

This puzzler is mathematical in nature. Imagine if you will, three gentlemen, Mr. Black, Mr. Brown and Mr. White, who so detest each other that they decided to resolve their differences with pistols. It's kind of like a duel—only a three-way duel. And unlike the gunfights of the old West, where the participants would simultaneously draw their guns and shoot at each other, these three gentlemen have come up with a rather more civilized approach.

Mr. White is the worst shot of the three and hits his target one time out of three. Mr. Brown is twice as good and hits his target two times out of three. Mr. Black is deadly. He never misses. Whomever he shoots at is a goner.

To even the odds a bit, Mr. White is given first shot. Mr. Brown is next, if he's still alive. He's followed by Mr. Black, if he's still alive.

They will continue shooting like this, in this order, until two of them are dead.
Here's the question: Mr. White is the first shooter. Remember, he's the worst shot. At whom should he aim his first shot to maximize his chances of surviving?

Clarification: At each stage of the duel the shooters can chose to shoot whomever they want or shoot in air/purposefully miss.

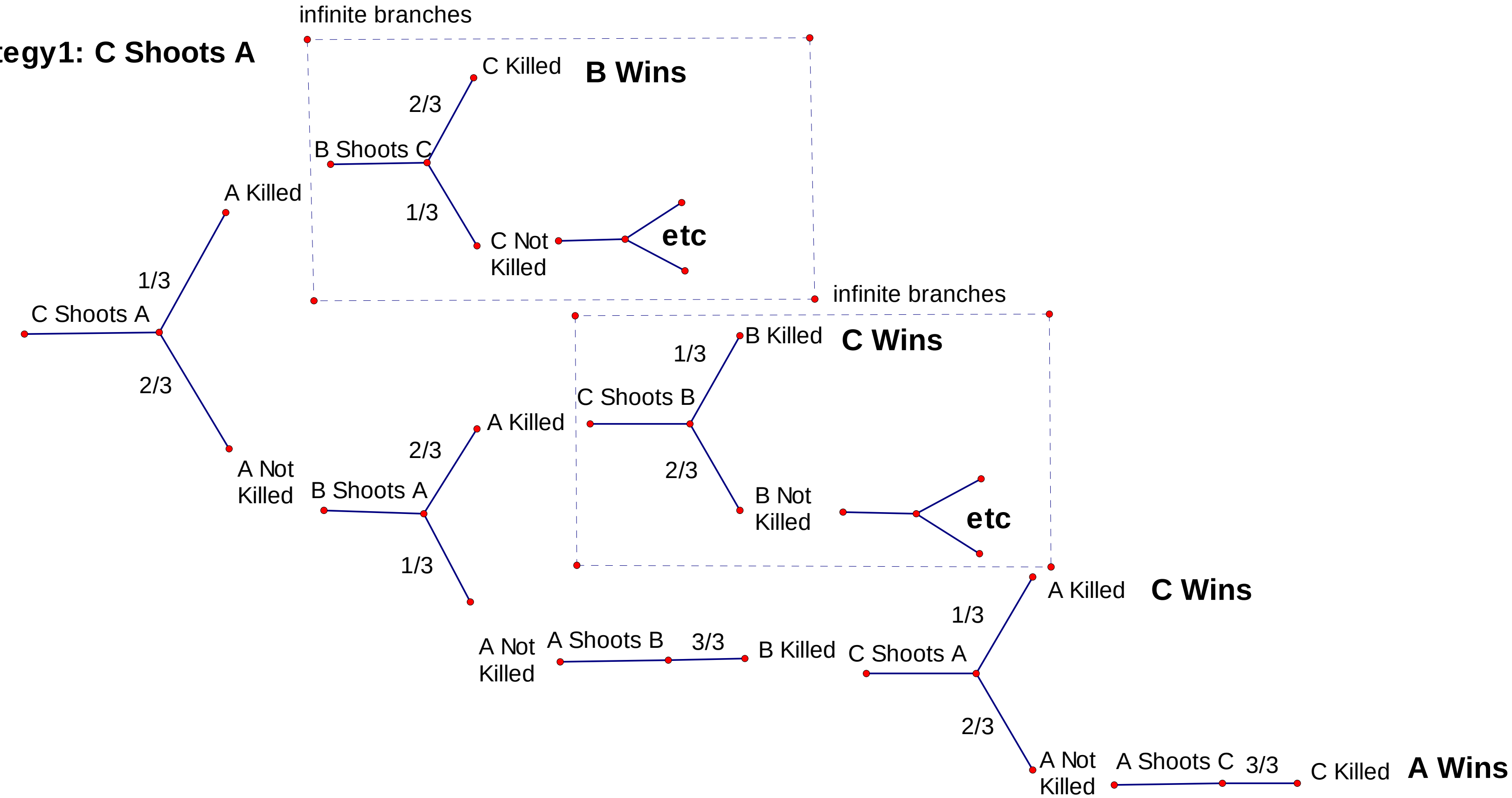
In this analysis Mr Black is called "A", Mr Brown "B" and Mr White "C" signifying the rank of their skills with the guns.

Assumption1: We will assume the shooters are rational thus they will make decisions on whom to shoot next based on logic rather than emotion, randomness etc. Thus we will assume that each shooter will seek to eliminate in a given round the opponent that is most "dangerous" from their point of view to remain alive unless again there is a rational reason hy not to do so. For example say it is A's turn to shoot and both B and C are alive. A will chose to shoot (and kill with certainty) B rather than C as B is more likely to kill him than C when left behind for the next round.

Assumption 2: We will assume that there is an infinite supply of bullets. If shooter A is eliminated in a round then there is no guarantee that the duel between B and C will terminate in a specified number of rounds.

Conclusion:
The work in the following pages indicate that if C shoots in the air/purposefully misses at the first round his probability of winning is 39.7% as compared to if he shoots at A at the first round where his probability of winning would be 31.2%. This is quite surprising as one might think it would be a waste to miss the chance to take A at round one. Also surprising is that in either case A has significantly low chance of winning 14.8% and 22.2% respectively.

Strategy1: C Shoots A



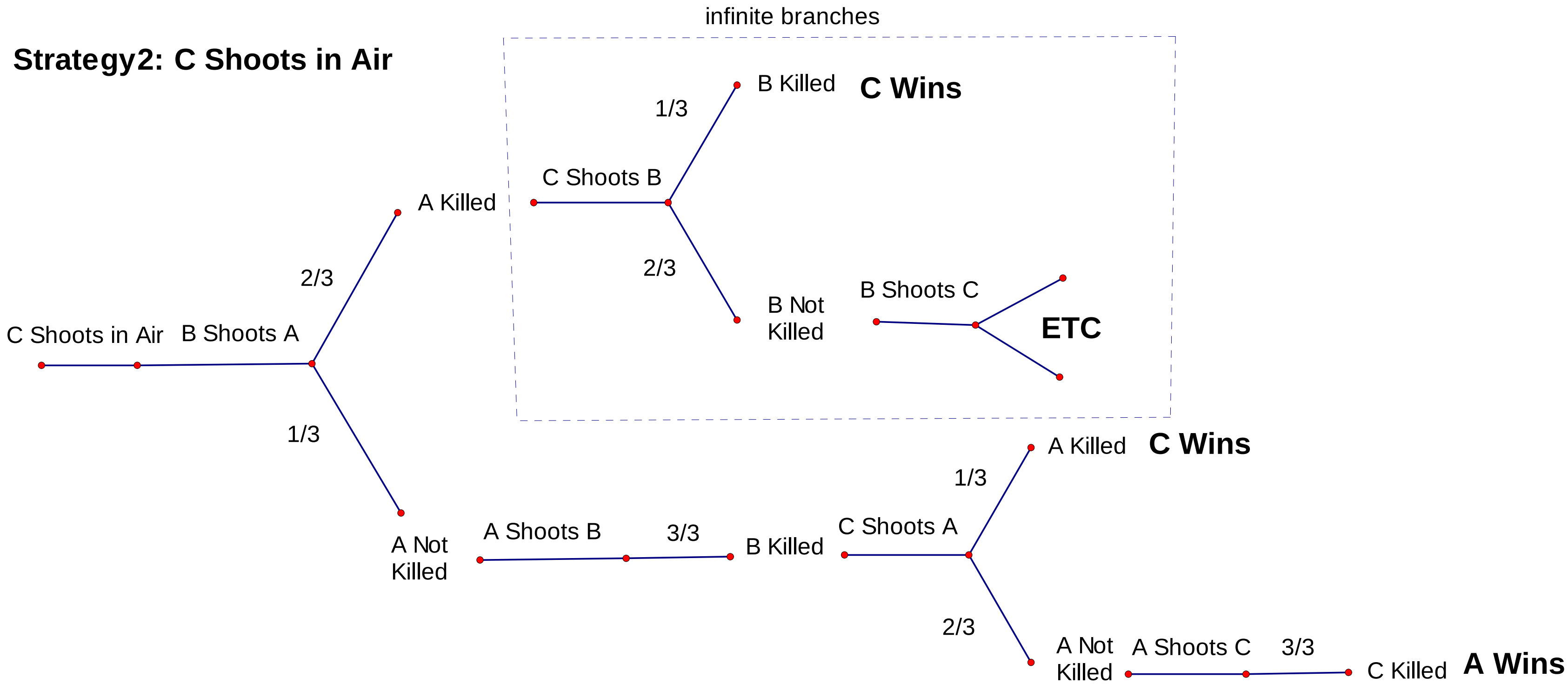
P(A wins)=(2/3)(1/3)(2/3)=28/189=0.148....

P(B wins)=(1/3)(6/7)+(2/3)(2/3)(4/7)=102/189=0.5397...

P(C wins)=(1/3)(1/7)+(2/3)(2/3)(3/7)+(2/3)(1/3)(1/3)=59/189=0.312....

See appendix for the case of the infinite branches

Strategy2: C Shoots in Air



P(A wins)=(1/3)(2/3)=14/63=0.222...

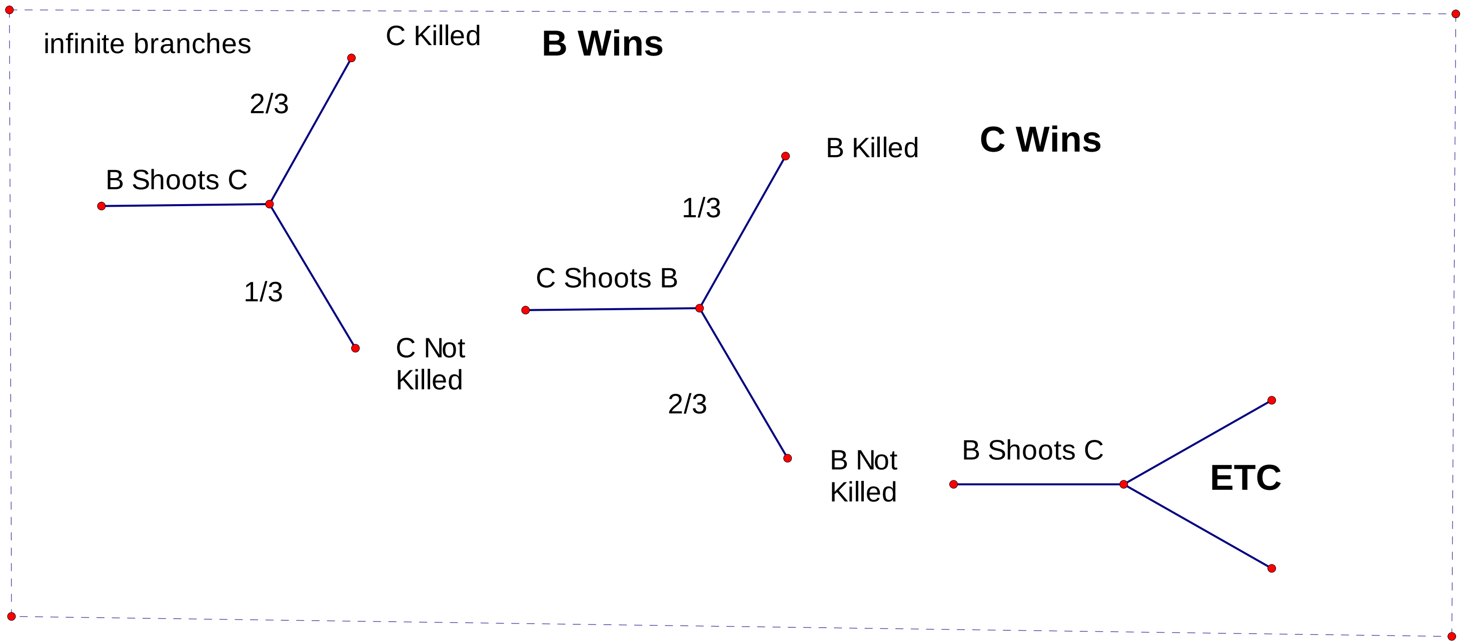
P(B wins)=(2/3)(4/7)=24/63=0.381....

P(C wins)=(2/3)(3/7)+(1/3)(1/3)=25/63=0.397...

See appendix for the case of the infinite branches

Appendix: Dealing with the infinite branched cases

Case1: B shoots at C first

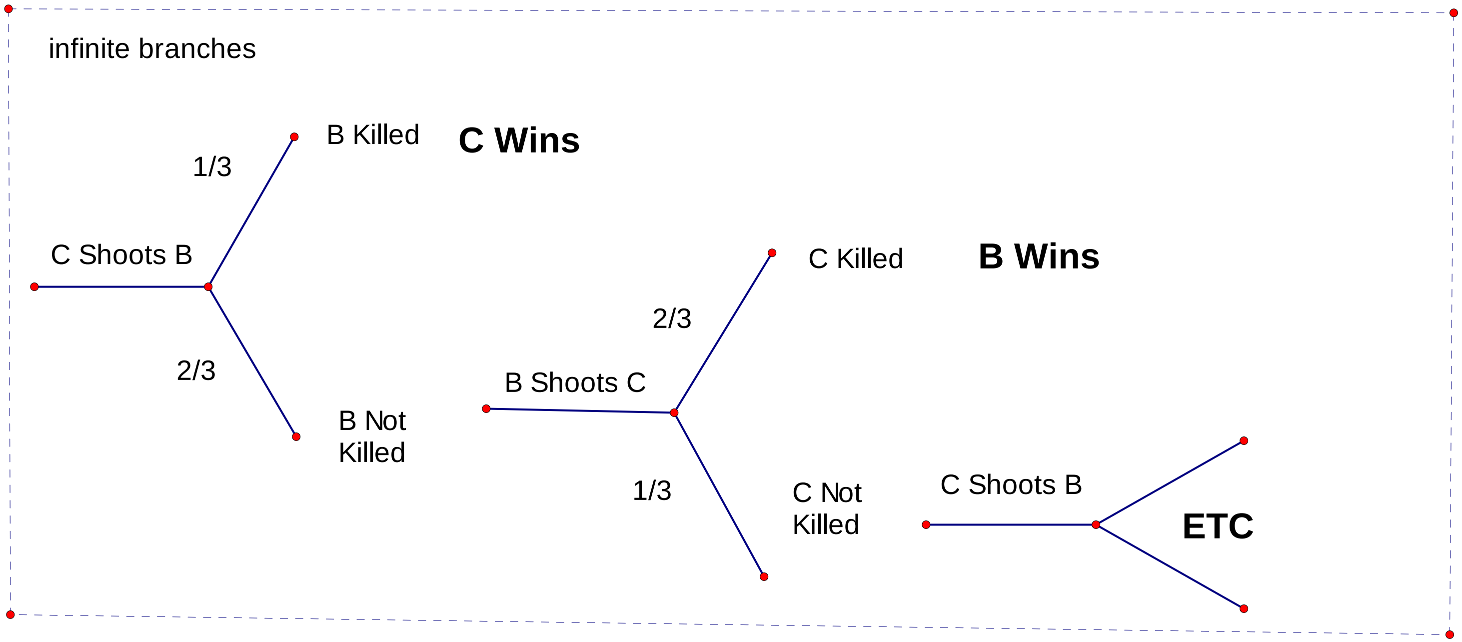


Using infinite geometic series sum

P(B wins)=(2/3)+(1/3)(2/3)(2/3)+......=(2/3)/(1-(2/9))=6/7

P(C wins)=(1/3)(1/3)+(1/3)(2/3)(1/3)(1/3)+....=(1/9)/(1-(2/9))=1/7

Case2: C shoots at B first



Using infinite geometic series sum

P(B Wins)=(2/3)(2/3)+(2/3)(1/3)(2/3)(2/3)+......=(4/9)/(1-(2/9))=4/7

P(C wins)=(1/3)+(2/3)(1/3)(1/3)(1/3)+.....=(1/3)/(1-(2/9))=3/7