A Tool for Student Inquiry

Students Explore Original Ideas with Sketchpad

By Selim Tezel, High School Mathematics Teacher, Concord Academy, Concord, MA

I used to teach in Istanbul, Turkey, at the high school division of Robert College. One year I had a wonderful student, Bilge Demirköz, who was a nontraditional learner. She had a style that was different than what the curriculum demanded. Before our work with Sketchpad, she never finished my tests on time. She was a very bright student, but not in a classical sense.

With the advent of technologies like Sketchpad, we started exploring various things. And one day, when we were studying conic sections, she asked a question that at the time really puzzled me because I had never thought about it myself: "What if, instead of two foci in an ellipse, you had three foci?"

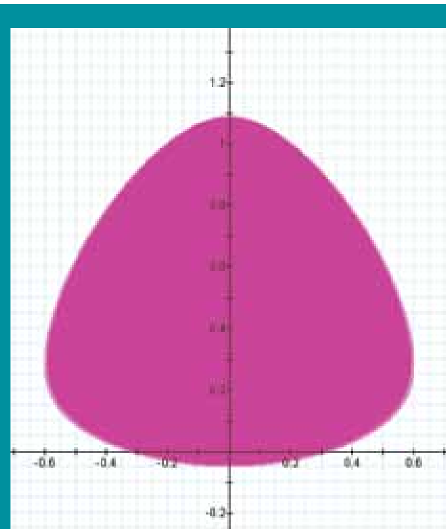
I didn't know the answer, but we realized that we had the right technologies to pose the question. So with the help of Sketchpad and another powerful computer program now called Graphing Calculator (from Pacific Tech, www.nucalc.com), we were able to actually visualize such constructions. And what my student found out later was that her question was originally posed by a 19th-century mathematician. The mathematician came to the point where he defined the mathematics my student investigated—he wrote the equations and so forth—but he could not visualize the equations because he did not have the means to do so.

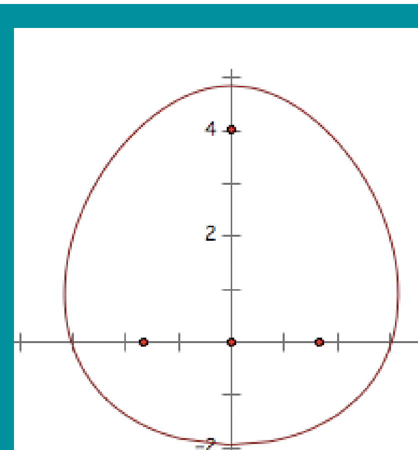
My student and I were very happy about this because we were able to finally make the next link in a historical chain of events that had come to a halt in the 19th century. My student wrote about her discoveries and became the youngest person ever published by the Turkish Mathematical Society. She eventually got accepted to MIT, where she graduated from the college and the master's program. She went on to study physics at Oxford.

In her case, and with many others, Sketchpad provided the environment for making an original conjecture and getting feedback on the idea. What students often miss in a mathematics classroom is appropriate feedback on their ideas. In other fields, computer science, for instance, if students make errors, the computer lets them know they've made errors. In a traditional mathematics classroom, it's hard to tell if you're on the right path or not. When you make things in Sketchpad, however, you get immediate feedback. This allows exploration to really speed up and become very rich.

My experience at Robert College wasn't my only exciting experience of this kind. One time when I was teaching about Archimedean tessellations (tessellations of mixed regular polygons) to high school freshmen and sophomores at a new school in the United States, one of the students did not follow my instructions. What happened was wonderful. He ended up creating a different type of ring-shaped tessellation from regular polygons.

I remember, though, my first reaction. I was a little bit upset that he didn't follow my instructions. But when I went home, I realized that he had generated a construction that was very rich mathematically. So with the help of two other freshmen, we developed the mathematics behind this discovery. We realized that the constructions related





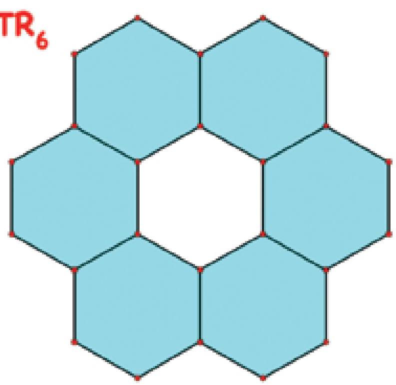
Robert College Student Bilge Demirkoz's three-foci curve drawn with Sketchpad

I do not believe that the possibility of high school freshmen publishing original scholarly work would be possible without the advent of technologies like Sketchpad.

geometry to number theory. Together, my students and I wrote a paper. Someday we hope to see it published.

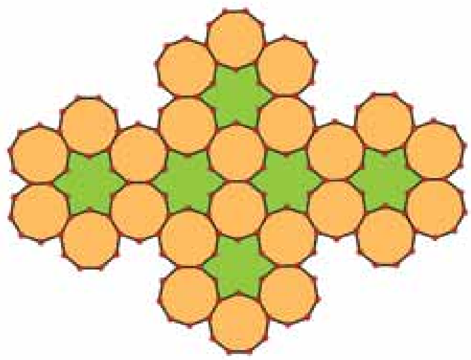
Sketchpad lets students explore their own ideas and get immediate feedback. It gives them confidence in their ideas. I do not believe that the possibility of high school freshmen publishing original scholarly work would be possible without the advent of technologies like Sketchpad. Sketchpad really is a tool for student inquiry. That's a big thing.

$_6TR_6$



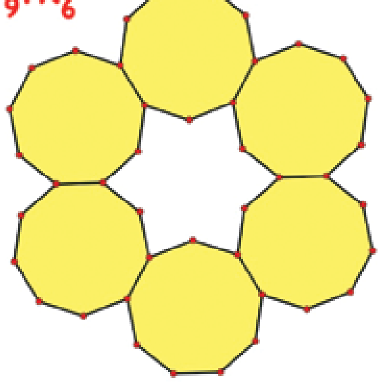
TESS-Rings:

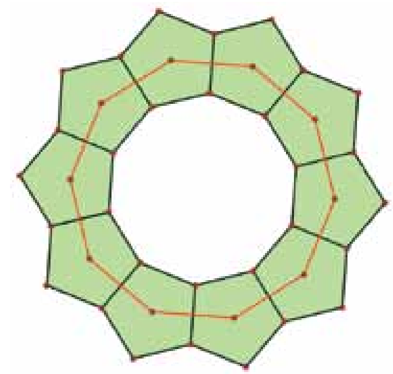
A Surprising Link Between Geometry and Number Theory



TESS-Rings were originally discovered in the spring of 2004 by Tony Diaz, when he was a sophomore at Concord Academy, during a Sketchpad explorations lab on the topic of Archimedean tessellations. While studying the nature of these structures, Tony came to realize that the rules of their geometric formation are deeply related to factoring of numbers, and thus the structures provide a beautiful link between geometry and number theory. We named these structures "TESS-Rings" to indicate their origins as related to TESSellations and to hint at who worked on them: Tony Diaz, Everett

$_9TR_6$





Wallace, Sylvie Lam, Selim Tezel. Tony's teacher, Selim Tezel, and his students have authored a paper on the subject of TESS-Rings and recommend that others may enjoy investigating the "Fundamental Question of TESS-Rings."

Definition: *A* TESS-Ring *is a two dimensional geometric structure formed by joining coplanar congruent regular polygons side by side until they form a closed ring-shaped figure, such as the one shown above. The centers of the regular polygons themselves create a regular polygon when joined by segments.* Definition: *The number of the sides of the regular polygon used to make the ring is called the* base *of the TESS-Ring and the number of regular polygons needed to form the ring is called the* period *of the TESS-Ring.*

The TESS-Ring shown above is made of ten regular pentagons joined side by side. Thus, the base of the ring is 5 and its period is 10. The polygon created by the centers is a regular decagon.

Notation: *A TESS-Ring formed by joining* k *regular* n*-gons is denoted by:* nTRk. *Thus* n *is the base and* k *is the period of the TESS-Ring. For example a TESS-Ring formed by ten regular pentagons is denoted:* 5TR10.

(Some more examples are at left.)

Fundamental Question of TESS-Rings: What is the relationship between the number of sides (*n*) of a regular polygon and the number of such regular polygons needed (*k*) to form a TESS-Ring? In other words, how does the period (*k*) of a TESS-Ring relate to its base (*n*)?